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# RESEARCH OF FRACTAL DIMENSION OF TIME SERIES OF GRAVIMETRIC OBSERVATIONS

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**Abstract.** The article is devoted to the analysis of time series of gravimetric observations using the value of fractal dimension as an indicator of persistence (stability) of the time series. Various numerical methods for assessing this indicator are examined and compared. Using real measuring data, quantitative estimates of the fractal dimension of nine trial time series of gravimetric observations are obtained for two components - the measured values of acceleration of gravity and root mean square deviations of the measurement errors. The obtained estimates are characterized by values from 1.12 to 1.44 for the first component and from 1.26 to 1.42 for the second component, which indicates the stable persistence (inertia) of these time series. The dependence of fractal dimension on the length of the series, the type of the trend-forming polynomial, the estimation method and its settings (using the methods of normalized range and minimum covering) are experimentally studied. Suggestions are given on the method for obtaining fractal dimension estimates and their practical application.

**Keywords:** gravimetric observations, time series, minimum coverage method, normalized range method, persistence, fractal dimension.

## Introduction

The solution of some problems of geodesy and geophysics is associated with the analysis of high-precision time series of gravimetric observations and there are already numerous works devoted to this subject (see, for example, [*Abramov, Koneshov, Chebrov,* 2016; *Abramov, Drobyshev, Koneshov,* 2013a; *Drobyshev, Koneshov, Nepoklonov,* 2016] and etc.). The processing capabilities of such series can be expanded by using fractal analysis as a tool that reflects the complexity of the process and allows diagnosing unstable states [*Kronover,* 2000; *Barton, Pointe,* 1995; *Feder,* 1988; *Hurst,* 1951; *Kantelhardt,* 2012]. The premise is the paradigm of the fractal (self-similar) structure of the set of stochastic processes of natural and man-made nature. The field of application of fractal analysis covers a number of geophysical applications, including the identification of observed geophysical processes, assessment of the quality and reliability of geophysical information, and forecasting the development of geophysical processes (phenomena). At the same time, fractal analysis is mainly used in the study of geotectonic structures, geodynamic and geomagnetic processes, seismic activity, meteorological phenomena, which are the subject of such works as [*Lyubushin,* 2007; *Imashev, Sychev,* 2017; *Dimri,* 2005; *Ranguelov, Ivanov,* 2017] and a number of others. In gravimetry, the use of fractal analysis today, apparently, is limited to the study of the spatial distribution of gravitational anomalies [*Barton, Pointe,* 1995; *Zhang, Featherstone,* 2000]. The use of a fractal approach to the processing and analysis of high-precision time series of gravimetric observations has not yet received a noticeable reflection in publications. The above served as the basis for the research discussed in this article.

## Formulation of the problem

High-precision time series of gravimetric observations in this work are understood as a set of discrete numerical indices of the observed signal - the acceleration of gravity, measured (registered) at successive, usually equidistant moments in time at a stationary gravimetric point. Observations can be carried out on the interval from several minutes to several days, with long-term observations - up to several months [*Abramov, Koneshov, Chebrov,* 2016; *Abramov, Drobyshev, Koneshov*, 2013б].

The time series $f(t),\ t \in \{t_1, t_2, ..., t_n \left(t_i < t_{i+1}, i = 1, ..., n-1\right)\}$ in the general case can be represented in the form $f(t) = \theta(t) + \delta(t) + \varepsilon(t)$, where $\theta(t)$ is a systematic component (trend); $\delta(t)$ is an irregular (pseudo-random, noise-like, chaotic) component; $\varepsilon(t)$ is a random noise; [*Amosov, Muller,* 2014].

The problem of identifying the systematic components of time series of gravimetric observations can be solved using approximating polynomials (algebraic, trigonometric). Algebraic polynomials describe, for example, the zero-point drift of a relative gravimeter. Trigonometric polynomials are used to describe periodic and cyclic components, in particular, tidal variations of the acceleration of gravity. It is common practice to determine the coefficients of the trend-forming polynomial using the least-squares method.

The problem of identifying random noise, which, as a rule, is a combination of various stochastic factors caused by the operation of sensors, recording devices and measuring system converters, can be solved using known methods and algorithms for filtering and smoothing time series.

The chaotic component, in a certain sense, occupies an intermediate position between the systematic and random components and largely reflects the specifics of the studied signal. The problem of identifying a chaotic component requires the use of special approaches and methods. One of them may be the method of fractal analysis, which is now widely used in various fields of scientific, technical and socio-economic activity [*Akhmetkhanov, Dubinin, Kuksova,* 2013; *Amosov, Muller,* 2014]. However, in comparison with other applications, the application of this method to the analysis of time series of gravimetric observations seems to be insufficiently studied for today.

In the method of fractal analysis, the chaotic nature of the process is estimated using the $D$ index, called the fractal dimension [*Mandelbrot,* 2002; *Pashchenko, Amosov, Muller,* 2015]. The $D$ value can be considered as a characteristic of the graph complexity, describing the distribution (development) of the process or phenomenon in space (time). Thus, analysis of data from areal and route gravimetric surveys showed that the larger the $D$ value, the higher the anomaly (roughness) of the surface or profile [*Zhang, Featherstone,* 2000].

When applied to time series, the fractal dimension serves as an indicator of the series persistence. Persistent series are characterized by the maintenance of the observed stable trend in combination with a relatively low level of noise. An alternating tendency in combination with a relatively high level of noise is characteristic of antipersistent series; after an increase in the function, it usually decreases, and after a decrease there is an increase. The following persistence criterion is used: the process is considered to be persistent when $1 \le D < 1.5$, antipersistent - when $1.5 < D \le 2$ and can be considered as a random walk when $D=1.5$ [*Akhmetkhanov, Dubinin, Kuksova,* 2013; *Osipov,* 2017].

Taking into account the noted circumstances, the task of this work was to study fractal approach to the analysis of time series of gravimetric observations with a quantitative estimation of fractal dimension on the example of real observations carried out under stationary conditions using modern high-precision relative gravimeters AutoGravCG-5 and CG-6. If gravimetric observations are carried out at specially equipped gravimetric stations using high-precision gravimetric equipment in a calm geophysical environment, it can be assumed that the series under consideration are likely to be persistent. However, this assumption requires experimental confirmation; the actual values of the fractality indices of the time series of gravimetric observations themselves are also of interest.

## Research method

The study was carried out in three stages, the first one was the choice of quantitative estimation methods of fractal dimension; the second is a computational experiment, including software development; the third is the analysis and interpretation of the obtained experimental data.

Currently, there are about two dozen of different methods for fractal dimension evaluation. In this paper we considered those of them, which, according to the authors, can be considered relevant in terms of time series analysis of gravimetric observations. Among them, the cell coverage method (Hausdorff – Kolmogorov method), the minimal coverage method, the normalized range method (the Hurst method), the standard deviation method, the Richardson's method, and the Fourier expansion method. In the numerical implementation of these methods, a construction of the form $\Omega_m = \{\omega_1, \omega_2, ..., \omega_m\}$ is used. It is the so-called uniform division of the interval $\omega = [t_1, t_n]$ with a step $\Delta t_m = (t_n - t_1)/m$, where $m$ is a given integer parameter.

In the method of cell coverage [*Kronover,* 2000], the $D$ index is determined on the basis of the ratio $N(\Delta t_m) \sim (1/\Delta t_m)^D$, where $N(\Delta t_m)$ is the number of cells of the cell coverage of the function $f(t)$ graph area on the partition $\Omega_m$, containing at least one point of the graph. The problem is reduced to calculating the slope of the straight line that approximates the dependence $N(\Delta t_m)$ on a double logarithmic scale [*Mitin,* 2013].

In the method of minimal coverage [*Dubovikov, Kryanev, Starchenko,* 2004], the $D$ index is determined using the amplitude variation $V_m = V(\Delta t_m)$ of the function $f(t)$ on the set $\Omega_m$:

$$V(\Delta t_m) = R(\omega_1) + ... + R(\omega_m), \quad (1)$$

where $R(\omega_i)$ is the range of the function $f(t)$ values on $\omega_i \in \Omega_m$ ($i$=1, 2, …, $m$). The right side of formula (1), multiplied by $\Delta t_m$, corresponds to the coverage of the function $f(t)$ graph by rectangles of the minimum area relative to the partition $\Omega_m$ (minimal coverage). The ration $V(\Delta t_m) \sim (\Delta t_m)^{1-D}$ gives

$$D = 1 + \mu, \quad (2)$$

where $\mu$ is the so-called fractal index, which definition is reduced to calculating the slope of the straight line that approximates the dependence $V(\Delta t_m)$ on a double logarithmic scale [*Mitin,* 2013].

In the normalized range method [*Hurst,* 1951; *Hurst, Black, Simaika,* 1965; *Mandelbrot,* 2002] $D$ is defined using the expression

$$D = 2 - H, \quad (3)$$

where $H$ is the so-called Hurst exponent, which is calculated based on the following formula scheme:

$$H = \frac{\ln(W(\omega))}{\ln(an)}; \tag{4}$$

$$W(\omega) = \frac{R(\omega)}{S(\omega)}; \tag{5}$$

$$R(\omega) = \max_{\omega}\left\{\sum_{t\in\omega}\left[f(t) - \bar{f}\right]\right\} - \min_{\omega}\left\{\sum_{t\in\omega}\left[f(t) - \bar{f}\right]\right\}; \tag{6}$$

$$S(\omega) = \sqrt{\frac{1}{n-1}\sum_{t\in\omega}\left[f(t) - \bar{f}\right]^2}. \tag{7}$$

Here $\bar{f}$, $R$ and $S$ are, respectively, the average value of the function $f(t)$, its largest range and standard deviation at a given interval; $W$ is the normalized range; $a$ is a given empirical coefficient, which value varies from 0.5 to $\pi/2$ in various works.

There are various methods for determining the Hurst exponent. The most commonly used method is where the $H$ value is determined by the slope of the straight line, which approximates in a double logarithmic scale the dependence of the arithmetic mean value of $\bar{W}_m$ of the normalized range on the elements of the partition $\Omega_m$ from step $\Delta t_m$, expressed accurate to a factor $a$ by the number of the function values on $\omega_i \in \Omega_m$, $i$=1, 2, …, $m$ [*Barabash, Maslovskaya*, 2010].

In the method of standard deviations and the Fourier expansion method, the $D$ index is calculated by the formula $D$=2–β, where β is the slope of a straight line approximating in a double logarithmic scale the dependence of the arithmetic mean value of the standard deviation $\bar{S}_m$ of the function $f(t)$ on the elements of the set $\Omega_m$ from step $\Delta t_m$ in the first case and dependence $c^2(\nu)$, where $c$ is an amplitude; ν is the frequency of the Fourier expansion of the function $f(t)$, in the second [*Barabash, Maslovskaya,* 2010].

In Richardson's method, $D$ is calculated according to the formula of the form (2) with the definition of the fractality index through the slope of the straight line approximating the double logarithmic dependence of the length $L$ of the equal-link broken line describing the function $f(t)$ graph on the length $r$ of its link. Besides, the value $L$ is determined by the expression

$$L(r) = N(r)r + \Delta r, \tag{8}$$

where $N$ is the number of links; $\Delta r$ is the remaining length [*Mandelbrot,* 2002; *Barabash, Maslovskaya*, 2010].

Each of the described methods for estimating the fractal dimension has its own advantages and disadvantages. Therefore, when choosing methods for the experimental study of time series of gravimetric observations, various criteria were taken into account - accuracy, reliability, flexibility, complexity of practical implementation, amount of calculations, practice of use. Taking into account all these criteria and the results of the analysis of other methods, described, in particular, in [*Kurdyukov et al.*, 2008; *Gaidukova,* 2016], two methods were selected for the computational experiment - the minimal coverage method (MCM) and the normalized range method (NRM).

It should be noted that the selected methods differ in approaches to assessing the fractal dimension: the first refers to the methods for the direct determination of the fractal dimension, the second - to the methods of its indirect assessment. However, the practical implementation of each method is formally reduced to calculating the second of the two coefficients of the straight line equation

$$y = \alpha + \beta x, \tag{9}$$

where to the discrete variables $x$ and $y$ correspond values $\ln(\Delta t_m)$ and $\ln(V_m)$ in MCM, $\ln(an/m)$ and $\ln(\bar{W}_m)$ in NRM.

The values of the initial coefficient β were calculated by the least-squares method under the condition of a minimum of the finite sum of deviations squares of $y_1, y_2, ..., y_k$ values of the variable *y* from the straight line (9) at the given values $x_1, x_2, ..., x_k$ of the variable *x*, where *k* is the number of the parameter *m* values.

The solution to this problem in an analytical form give expressions

$$\beta = \frac{\sum_{i=1}^{k}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{k}(x_i - \bar{x})^2}, \quad (10)$$

$$\sigma_\beta = \sqrt{\frac{1}{k-2}\left[\frac{(y_1 - \bar{y})^2 + ... + (y_k - \bar{y})^2}{(x_1 - \bar{x})^2 + ... + (x_k - \bar{x})^2} - \beta^2\right]}, \quad (11)$$

where $\bar{x}$ and $\bar{y}$ are, respectively, the arithmetic mean values $x_i$ and $y_i$ ($i$=1, 2,…, $k$); $\sigma_\beta$ is the standard deviation of the error in determining the coefficient β [*Popov, Nozik*, 2019].

In the course of the experiment, the dependences of the fractal dimension estimates on a number of factors that as was assumed, can influence the result were investigated. These factors include the chosen method for calculating the fractal dimension; degree *q* of the polynomial $P_q(t)$, used to identify the trend; number of time series values; type of analyzed information. When investigating the dependence of the estimates on the first of the named factors, the above-mentioned methods NRM and MCM were used with varying of their settings, determined by the number *k* of variants of the segment $\omega = [t_1, t_n]$ uniform partition and the number $n_i$ of the function *f*(*t*) values on the intervals of the *i*-th partition ($i$=1, 2,…, $k$); for the second factor, various trend-forming polynomials $P_q(t)$ were used for the values of *q* - algebraic polynomials of degree *q*=1, 3, 5, 9; for the third factor were used time series of gravimetric observations, including different number of measurements (from several thousands to several tens of thousands).

The dependence of the fractal dimension estimates on the type of used information (the fourth of the named factors) was investigated involving various components of time series of gravimetric observations - not only the acceleration of gravity (AG) was analyzed, but also the root mean square deviations of measurement errors (RMSD) as another informative parameter.

Optimization of settings is an open issue. The value $n_i$ can vary from $n_i$=2 to $n_i$=[$n$/2], where [ ] is the integral part of a number, however, enumeration of all options can be associated with a large amount of computation. In this work, the $n_i$ values were set according to Table. 1. In addition, two options were considered for each method: first – only $n_1, n_2, ..., n_8$ $(k = k_1 = 8)$; second – all ten values $(k = k_2 = 10)$. The value of the factor *a* in all variants of using the normalized range method was taken equal to 1.

**Table 1.** Setting parameters for assessing fractal dimension

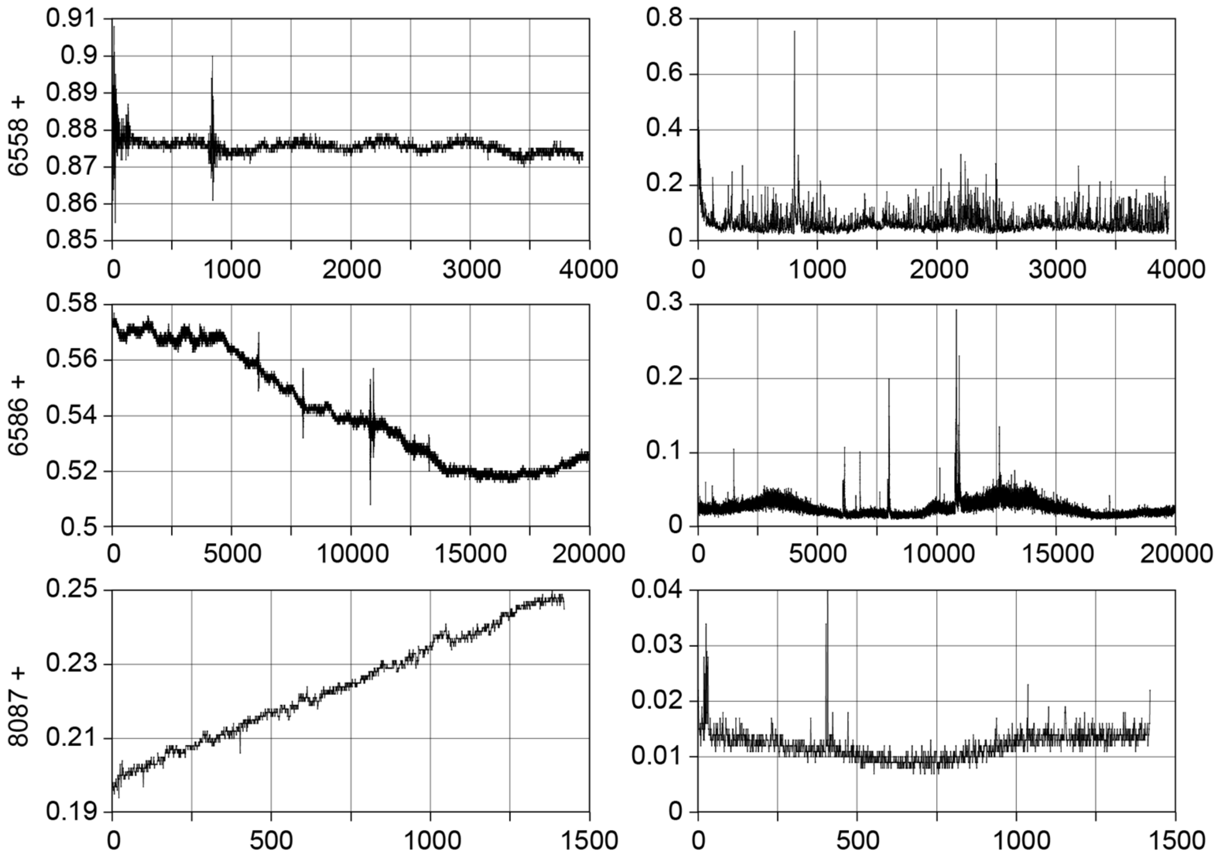
| Method | $n_1$ | $n_2$ | $n_3$ | $n_4$ | $n_5$ | $n_6$ | $n_7$ | $n_8$ | $n_9$ | $n_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| MCM | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| NRM | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

The initial information was the results of real gravimetric observations carried out at the fundamental gravimetric station "Dolgoe Ledovo" (Moscow region), the gravimetric observatory "Zapolskoe" (Vladimir region) and in the gravimetric laboratory of MIIGAiK [*Koneshov, Abramov, Dorozhkov,* 2010; *Yuzefovich, Yuzefovich,* 2015]. Nine gravimetric series with a discreteness of observations 60 s were used for the study (Table 2).

**Table 2.** General characteristics of the initial data of gravimetric observations that were used in the study

<table>
<tr><th>Place of conducting</th><th>Series</th><th>Type of the device</th><th>Start date</th><th>End date</th><th>Amount of observations</th></tr>
<tr><td rowspan="3">“Dolgoe Ledovo”</td><td>1</td><td rowspan="7">CG-5 AutoGrav</td><td>06.05.2015</td><td>12.05.2015</td><td>8216</td></tr>
<tr><td>2</td><td>12.05.2015</td><td>15.05.2015</td><td>3944</td></tr>
<tr><td>3</td><td>17.07.2015</td><td>20.07.2015</td><td>3991</td></tr>
<tr><td rowspan="4">“ Zapolskoe”</td><td>4</td><td>15.05.2015</td><td>29.05.2015</td><td>19951</td></tr>
<tr><td>5</td><td>29.05.2015</td><td>30.05.2015</td><td>1700</td></tr>
<tr><td>6</td><td>30.05.2015</td><td>03.07.2015</td><td>48287</td></tr>
<tr><td>7</td><td>03.07.2015</td><td>17.07.2015</td><td>20654</td></tr>
<tr><td rowspan="2">MIIGAiK</td><td>8</td><td rowspan="2">CG-6 AutoGrav</td><td>02.07.2018</td><td>03.07.2018</td><td>1440</td></tr>
<tr><td>9</td><td>04.07.2018</td><td>05.07.2018</td><td>1420</td></tr>
</table>

The figure below shows the graphs of the dependence of the measured values of the acceleration of gravity (AG) and root mean square deviations (RMSD) on the observation time, typical for the time series under study.




Time dependences of measured values of the gravitational acceleration (*on the left*, in the instrument scale reading; the numbers near the vertical axes are the integer part of the readings) and root mean square deviations (*on the right*), constructed for time series of gravimetric observations at "Dolgoe Ledovo" station, row 2 (*at the top*), Zapolskoye observatory, row 4 (*in the center*) and at MIIGAiK laboratory, row 9 (*at the bottom*). On the horizontal axes are the numbers of the observations in rows showing the time of observation. Characteristics of the used data, see Table. 2

The graphs above illustrate the extent to which the structure and behavior of the series of measured values of the acceleration of gravity and their root mean square deviations can vary depending on the observation point.

The analysis of the studied time series showed that in the values of the acceleration of gravity, almost all of them have a rather noticeable linear trend, which in most cases (six out of nine) is upward, in two cases - downward. In the values of the root-mean-square deviation, such a trend is practically absent, but the oscillations of the RMSD look sharper compared to the oscillations in the measured values of AG at more pronounced amplitudes. In general, the results of observations at the Dolgoe Ledovo point have the lowest stability in time. This applies to both trend characteristics and oscillation amplitudes, especially for RMSD.

## Computational experiment results and their discussion

The integral indices $\bar{D}$ and $R_D$ of the fractal dimension estimates, obtained by the described method in the computational experiment, are given in Table 3, 4. These are, respectively, the arithmetic mean and the range of changes in the $D$ value estimates for various values of the degree $q$ of the trend-forming polynomial $P_q(t), t \in \omega$. The $\Delta D$ value is an estimate of the standard deviation of the fractal dimension according to formula (11).

As it can be seen, $\bar{D}$ is characterized by the following values: for AG (see Table 3) – from 1.12 to 1.44 (average – 1.29); for RMSD (see Table 4) – from 1.26 дtoo 1.42 (average – 1.34), and values of $\Delta D$, as a rule, are in the range from 0.01 to 0.03. The obtained estimates of the fractal dimension are consistent with the statement given in [*Kuznetsov*, 2011] that for terrestrial processes the Hurst exponent is approximately 0.73±0.09, provided that seasonal variations are sufficiently complete eliminated.

**Table 3.** Estimates of the fractal dimension of the acceleration of gravity values (AG)

| Series | NRM | | | | MCM | | | |
|---|---|---|---|---|---|---|---|---|
| | $\bar{D}\pm\Delta D$ | | $R_D\times10^3$ | | $\bar{D}\pm\Delta D$ | | $R_D\times10^3$ | |
| | $k_1$ | $k_2$ | $k_1$ | $k_2$ | $k_1$ | $k_2$ | $k_1$ | $k_2$ |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 1.254±0.015 | 1.240±0.014 | 4 | 3 | 1.380±0.027 | 1.413±0.025 | 0 | 0 |
| 2 | 1.250±0.025 | 1.221±0.025 | 6 | 9 | 1.407±0.026 | 1.437±0.024 | 1 | 1 |
| 3 | 1.250±0.014 | 1.221±0.022 | 27 | 43 | 1.373±0.029 | 1.403±0.028 | 0 | 0 |
| 4 | 1.231±0.017 | 1.210±0.017 | 2 | 1 | 1.379±0.029 | 1.411±0.026 | 0 | 0 |
| 5 | 1.205±0.029 | 1.191±0.019 | 28 | 22 | 1.342±0.018 | 1.365±0.026 | 1 | 4 |
| 6 | 1.191±0.019 | 1.143±0.021 | 122 | 144 | 1.365±0.026 | 1.395±0.024 | 4 | 5 |
| 7 | 1.209±0.012 | 1.190±0.014 | 8 | 15 | 1.360±0.027 | 1.393±0.025 | 0 | 1 |
| 8 | 1.250±0.014 | 1.232±0.016 | 17 | 13 | 1.363±0.022 | 1.390±0.026 | 0 | 0 |
| 9 | 1.120±0.015 | 1.114±0.012 | 12 | 11 | 1.337±0.024 | 1.369±0.024 | 1 | 1 |

**Table 4.** Estimates of the fractal dimension of the root mean square deviation values (RMSD)

| Series | NRM | | | | MCM | | | |
|---|---|---|---|---|---|---|---|---|
| | $\bar{D}\pm\Delta D$ | | $R_D$ | | $\bar{D}\pm\Delta D$ | | $R_D$ | |
| | $k_1$ | $k_2$ | $k_1$ | $k_2$ | $k_1$ | $k_2$ | $k_1$ | $k_2$ |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 1.305±0.006 | 1.307±0.005 | 0 | 1 | 1.256±0.028 | 1.287±0.026 | 0 | 0 |
| 2 | 1.308±0.011 | 1.304±0.009 | 1 | 1 | 1.256±0.028 | 1.292±0.027 | 0 | 1 |
| 3 | 1.310±0.008 | 1.310±0.006 | 1 | 1 | 1.263±0.025 | 1.286±0.021 | 0 | 0 |
| 4 | 1.363±0.007 | 1.357±0.006 | 2 | 2 | 1.393±0.026 | 1.421±0.023 | 0 | 0 |
| 5 | 1.335±0.013 | 1.314±0.018 | 63 | 61 | 1.380±0.027 | 1.412±0.025 | 10 | 7 |
| 6 | 1.368±0.002 | 1.364±0.003 | 0 | 0 | 1.368±0.025 | 1.414±0.024 | 0 | 0 |
| 7 | 1.351±0.006 | 1.347±0.005 | 2 | 4 | 1.371±0.023 | 1.399±0.022 | 0 | 0 |
| 8 | 1.372±0.011 | 1.366±0.012 | 32 | 54 | 1.360±0.021 | 1.389±0.021 | 2 | 2 |
| 9 | 1.363±0.012 | 1.362±0.011 | 41 | 62 | 1.388±0.027 | 1.417±0.025 | 1 | 0 |

Judging by the results, all nine studied time series for both components are stably characterized by the fractal dimension $D<1.5$. This means that almost all series, regardless of their duration in time and place of observation, can be considered persistent. The persistence property, as noted above, means the preservation of the observed trend in combination with a relatively low level of noise. The distinctive properties of persistent processes include self-similarity (independence from the time scale) and predetermined behavior, which is natural for high-precision gravimetric observations. At the same time, the obtained estimates of the fractal dimension vary within the above ranges. These fluctuations take place both within the same series of gravimetric observations and between different series. Analysis of the nature of empirical values oscillations of the fractal dimension indicates that they are mainly due to the specifics of the method of their calculation (NRM, MCM) and the type of the analyzed component (AG, RMSD). The dependence of the obtained estimates on the remaining factors (the internal settings of each individual method, given in Table. 1, the degree of the trend-forming polynomial, the number of observations in the series) is also evident, but to a lesser extent.

The influence of the settings of the method for calculating the fractal dimension is characterized by discrepancies in the estimates of the same name for the $\bar{D}$ value with an increase in the number of variants *k* of the uniform partition of the time series from $k_1$=8 to $k_2$=10. Analysis of Table 3, 4 show that this factor acts in different directions: for NRM, it reduces the $\bar{D}$ value according to AG by an average of 0.03 and according to RMSD by less than 0.01; for MCM - in both cases (according to AG as well as according to RMSD), this factor increases $\bar{D}$ by an average of 0.03.

It was also experimentally established that the estimate of the fractal dimension can be influenced not only by the number *k* of variants of a uniform partition, but also by the amount of initial data on the interval of such a partition - specific values $n_1$, $n_2$, …, $n_k$.

The dependence of the estimates on the type of the trend-forming polynomial, characterized by $R_D$ index is more pronounced for NRM, while the MCM response seems to be weaker. This is probably due to the peculiarities of the numerical implementation of the two methods. First of all, let us note the differences in the intervals for splitting the series (see Table 2). In the MCM, they are shorter, which was due to the desire for a more complete record of the irregularity of the function *f*(*t*) graph. As a result, the trend nonlinearity manifests itself weaker than on the longer intervals of NRM, which choice, in turn, was due to the desire for a more reliable disclosure of the self-similarity effect of the analyzed time series. As a result, NRM showed a higher sensitivity to the choice of the trend-forming polynomial degree. At the same time, it was established that the influence of this factor may be different depending on the specific series. The greatest discrepancies in the estimates of the fractal dimension at different *q* values have series 3, 6 for AG and series 5, 9 for RMSD. However, there are practically no regularities - neither by the place of obtaining the series, nor by the number of observations, nor by the type of analyzed information.

Empirical dependences of the *D*(*q*) type can be considered as additional information when justifying the choice of the trend-forming polynomial degree in the process of eliminating the influence of systematic errors of the studied time series. Decreased (in this case, close to 1.0) estimates of *D* may indicate an insufficiently complete elimination of the systematic component, which residual part is superimposed on the chaotic component. On the contrary, increased ($D\approx1.5$) estimates may indicate an excessively high degree of the used trend-forming polynomial, which leads to the appearance of false oscillations

The noted features of the numerical implementation of two methods probably influenced the difference between the estimates of *D* values obtained with their use. In Table 3, 4, it can be seen that the *D* values obtained using NRM are in 100% of cases for AG and in 50% of cases for RMSD are below the corresponding estimates obtained using the MCM. If we proceed from the average level, then, in general, the use of MCM demonstrates its tendency to overestimate the persistence degree of time series of gravimetric observations, and the use of NRM shows its tendency to underestimate.

The reaction of *D* estimates to the length of the series was revealed, i.e. to the number of the function *f*(*t*) values over the observation interval. However, it is not necessary to speak of any regularity in this case, as evidenced by the relatively small values of the correlation coefficients of the $\bar{D}$ and *n* values: from –0.36 to 0.10 for AG; from 0.29 to 0.43 for RMSD.

The correlatability of fractal indices of the analyzed time series obtained by two different methods (NRM, MCM) and for two different components (AG, RMSD) given in columns 2, 3, 6, 7 of Table 3, 4 was traced. Statistical calculations showed the following values of the corresponding correlation coefficients: for AG series - from 0.58 to 0.99; for RMSD series - from 0.79 to 0.99; between series of AG and RMSD - from –0.70 to –0.40; between the estimates of NRM and MCM - from 0.58 to 0.73 for AG series and from 0.79 to 0.90 for RMSD series.

Thus, with all possible differences in the estimates of the fractal dimension, depending on the type of the studied component of the output signal of the gravimeter (AG, RMSD) and the method of obtaining, these estimates in the statistical sense can be quite strongly correlated both in the positive and negative area. A positive correlation is consistently demonstrated by estimates obtained by various methods for each individual component of the series. Negative correlation takes place in all variants of comparing fractal dimension values for different components of the output signal, which is due to the noticeable predominance of a downward trend in AG series estimation and an upward trend in RMSD series estimation.

Each of the two components of the analyzed series is characterized by its own values of the *D* index, which differ to some extent from these indices for the other component. Comparison of these indices at different time intervals allows to consider RMSD value not only as a characteristic of the measurement quality, but also as a possible source of additional information on the temporal instability of the acceleration measured with a gravimeter. Note that the possibility of using RMSD as an informative parameter characterizing the influence of environmental factors (seismic, atmospheric) has already been discussed earlier (see, for example, [*Abramov, Drobyshev, Koneshov,* 2013; *Drobyshev, Koneshov,* 2013]).

The obtained experimental confirmation of the dependence of the fractal dimension empirical estimates of different components of the analyzed series on the estimation method and the scheme of its numerical implementation gives reason to suppose that it is reasonable to use an integrated approach. It should be noted that it was already pointed out in [*Kirichenko, Chalaya,* 2014]. The goal in this case is to increase the reliability and validity of estimates of fractal dimension.

The essence of the integrated approach is to determine the final index *D* by averaging partial estimates obtained using, firstly, at least two different methods, one of which (in this case - NRM) uses formula (4), the other (in this case - MCM) - formula (2). Secondly, each method requires the consideration of several variants of its implementation, which are determined, in particular, by the number of used variants of the uniform partition of the observation interval and the number of values of the studied function on the elements of the partition. Appropriately determined weighting coefficients may be used by averaging since not only the *D* estimates vary, but also their accuracy characteristics (see Tables 3, 4).

## Conclusions

In work on theoretical and experimental material was investigated the application of fractal analysis methods to assess persistence of time series of gravimetric observations and were obtained results that allow to draw the following conclusions.

1. Experimental data confirm the stated assumption about the persistence of time series of gravimetric observations obtained under stationary conditions at specially equipped gravimetric stations using high-precision gravimetric equipment.

2. Empirical estimates of the fractal dimension depend on various factors, including the method used to estimate it and the type of trend-forming polynomial. To increase the reliability of determining the fractal dimension, it is advisable to use various methods for assessing the fractal dimension with the output of the final result for a specific time series by averaging the estimates calculated by each individual method, taking into account the errors in determining these estimates from the measurement results.

3. Estimates of the fractal dimension as an indicator of the persistence of time series of gravimetric observations can be used to justify the choice of the optimal degree of the trend-

forming polynomial according to the criterion of achieving stable values by these estimates in the interval from 1 to 2.

4. One of the possible ways to increase the reliability of determining the fractal dimension for time series of gravimetric observations is averaging the estimates of the fractal dimension of gravimetric observations array obtained by various methods within the framework of an integrated approach implementation with varying its internal settings for each method, including the parameters of the uniform partition of the observation interval.

5. It is advisable to continue the improvement of the methodology for studying the fractal characteristics of time series of gravimetric observations, which possible directions are the rational expansion of the composition of methods used to estimate the fractal dimension, as well as optimization of the variable internal settings and setting parameters of these methods, including the parameters of uniform partitioning of the observation interval.

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