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RELATIONSHIP BETWEEN PRECEDING SEISMICITY AND THE PROBABILITY OF STRONG AFTERSHOCK OCCURRENCE

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Abstract. The paper considers the relationship of seismicity, preceding the main shock, with the emergence probability of strong aftershocks in the future series. (Strong aftershock in the sense of Bath's law means that its magnitude not lower than the average difference of magnitudes of the strongest aftershocks and their mainshocks.) A hypothesis of the research – strong aftershocks are more likely to occur due to mainshocks occurred in places of high background seismicity. Testing the hypothesis at the global level was carried out using ANSS ComCat earthquake catalog US Geological Survey, at the regional level – using earthquake catalogs provided by Geophysical Service of the Russian Academy of Sciences for seismic regions of Russia (Kamchatka and the Kuril Islands, Baikal and Transbaikalia, the North Caucasus). We tested several functions that characterize previous background seismic activity relative to the mainshock. The values of the functions were considered as a possible precursor or anti-precursor of a strong aftershock. The effectiveness of the precursor (anti-precursor) was evaluated by a specially developed criterion, representing the ratio of the sum of all successful forecasts to the number of all unsuccessful forecasts. The value at which the maximum efficiency is achieved was taken as a threshold. The value of the previous activity greater or less than the threshold was considered as a precursor or anti-precursor of a strong aftershock, respectively. As a result, the hypothesis of the study was confirmed at the global and regional levels, regardless of the method of measuring previous seismic activity. The most informative characteristic of activity is the ratio of the accumulated seismic moment of background earthquakes preceding the main shock to the time of the main shock, normalized to the area of the circle bounding the background seismicity region. The probability of expected repeated shocks was estimated using the Reasenberg–Jones model depending on time and magnitude. We estimated the model parameters for the Earth and seismically active regions of Russia both with and without preceding seismicity. Comparison of theoretical and model values of the probability of occurrence of at least one strong aftershock at different time intervals showed that the model corresponds well with the initial data. Using the probability gain, we shown that Reasenberg–Jones model, which takes into account preceding background seismicity, is more preferable than the model without it.

Keywords: seismic activity, background earthquakes, precursor, aftershocks, probability.

Introduction

The paper considers the relationship between the occurrence of a strong aftershock and seismicity before the main shock. A strong shock is regarded as an aftershock with magnitude $M \geq E[M_1]$, where $E[M_1]$ is the average magnitude difference of the strongest aftershocks and main shocks calculated from data for the Earth as a whole and for three seismically dangerous regions of Russia - Kamchatka and the Kuril Islands, Baikal and Transbaikalia, the North Caucasus. According to the Bath's law [*Bath*, 1965], this average value does not depend on the magnitude of the main shock, and its specific value is determined by the used magnitude scale.

The objective of the study is to test the author's hypothesis, according to which the occurrence of strong aftershocks is more likely after the main shocks confined to places with high background seismicity. The authors used data from the *ANSS ComCat* global catalog of earthquakes of the US Geological Survey and data from regional catalogs of the Federal Research Center "Geophysical Survey of the Russian Academy of Sciences" (GS RAS).

A number of simple functions describing seismic activity relative to the main shock were tested. In particular, we analyzed the ratio of the sums of the lengths of earthquake faults before the main shock to the length of the main shock faults; the ratio of the total Benioff strain from earthquakes before the main shock to the strain resulting from the main shock; the ratio of the sum of the areas of earthquake fault before the main shock to the area of the main shock fault; the ratio of the total scalar moment of earthquakes before the main shock to the scalar moment of the main shock. The problem of partitioning the set of values of the relative preceding seismic activity in terms of the precursor (anti-precursor) of a strong aftershock was considered.

The hypothesis was tested by solving the binary forecast problem: using the value of seismic activity before the main shock, the authors determined (predicted) whether a strong aftershock would occur or not. Successful predictions were cases when the value of the preceding seismic activity exceeded the threshold value for the main shock accompanied by a strong aftershock (*true positive*), or when this value was lower than the threshold value for the main shock, which was not followed by a strong aftershock (*false negative*). Unsuccessful forecasts included cases when the value of the preceding seismic activity was higher than the threshold for the main shock without a strong aftershock (*false positive*) or when the activity value was below the threshold for the main shock, followed by a strong aftershock (*true negative*).

To measure the effectiveness of binary forecasts, the authors proposed a new criterion *SFR* (***S**uccess **F**ailure **R**atio*) – the ratio of the total number of "successes" to the total number of "failures". The threshold value of preceding seismicity is determined by *SFR* maximization. The novelty of the criterion lies in the simultaneous consideration of all possible "successes" and "failures", which eliminates the need to write two separate expressions for gain in probability and for the *ROC*-diagram (***R**eceiver **O**perating **C**haracteristic*) (for more details on *ROC*-diagrams, see below).

To determine the probability of a strong aftershock, the model of P. Reasenberg and L. Jones [*Reasenberg, Jones,* 1989], representing the aftershock process by superposition of the Gutenberg – Richter laws [*Gutenberg, Richter,* 1954] and Omori – Utsu [*Utsu*, 1961], was used. Besides, the model parameters were estimated for the Earth as a whole and separately for a number of Russian regions.

To illustrate the effect of high preceding background seismicity on the occurrence of repeated shocks, the authors calculated the probability of the occurrence of aftershocks with magnitudes not lower than the magnitude of the main shock M_m and M_m–1 depending on time.

The results of the performed research by the authors are of great practical importance, since they allow us to evaluate the possibility of repeated strong shocks that occur after strong earthquakes.

Initial data

As it was mentioned above, as initial data for the Earth as a whole, data from the *ANSS ComCat* global catalog of the US Geological Survey[1] was used. The seismicity of the considered seismically dangerous regions of Russia was taken in accordance with the regional catalogs of the Federal Research Center of the "United Geophysical Service of the RAS" and its branches. So, for Kamchatka and the Kuril Islands, the catalog of earthquakes of Kamchatka and the Komandorskiye Islands of the Kamchatka branch of the GS RAS[2] was used; for Baikal

[1] *URL: https://earthquake.usgs.gov/data/comcat/* (accessed 25.12.2018).
[2] *URL*: *http://www.emsd.ru/sdis/earthquake/catalogue/catalogue.php* (accessed 25.12.2018).

and Transbaikalia - the catalog of earthquakes of Baikal and Transbaikalia of the Irkutsk branch of the GS RAS[1]. The catalog of Caucasian earthquakes of the GS RAS used for the North Caucasus [*Gabsatarova, Borisov*, 2017, 2018] was supplemented by data from the Caucasus seismological bulletin for 1971-1986 [Seysmologicheskiy..., 1990], from which explosions were excluded. The main shocks and aftershocks were allocated by the method of G.M. Molchan and O.E. Dmitrieva [*Molchan, Dmitrieva*, 1992], implemented in the program of V.B. Smirnov [2009].

For the Earth as a whole as main shocks were considered earthquakes with $M \geq 6.5$ and focal depth ≤ 200 km (718 such series were identified in the period from 1975 to October 2017); for Kamchatka and the Kuril Islands – earthquakes with $M_L \geq 6$ and focal depth ≤ 200 km (69 series were identified in the period from 1962 to October 2017); for Baikal and Transbaikalia – earthquakes with $M \geq 5.5$ (36 series were identified in the period of 1960–2015); for the North Caucasus – earthquakes with $M \geq 5$ (42 series were identified in the period from 1960 to May 2016). It should be noted that since there are no main shocks with focal depths of more than 200 km for Baikal and Caucasus region, this condition was not used in the selection of series.

Background earthquakes were distinguished by the "nearest neighbor" method [*Zaliapin, Benzion,* 2016], which is based on the calculation of the proximity function between the current earthquake and preceding events, depending on the parameters of the seismic regime – the *b*-values and fractal dimension. The named method was widely used due to the relative simplicity and high visualization of the separation of background and grouped (swarms, foreshocks, aftershocks) events. And although, in our opinion, compared with the Molchan – Dmitrieva method, this method is worse for identifying series of aftershocks, in the case of allocating background earthquakes after its application, there are practically no grouped events.

The relationship of seismicity preceding the main shock with the occurrence of a strong aftershock

The spatial region of a circle shape with the center coinciding with the epicenter of the main shock was considered; circle radius *R* is proportional to the length of the fault and is determined as

$$R\,[km] = cf \cdot 10^{M_m/2}, \tag{1}$$

where *cf* is the parameter to be determined; M_m is the magnitude of the main shock [*Baranov, Shebalin,* 2018a].

We analyzed the background earthquakes for *T* years before the main shock, allocated by the "nearest neighbor" method from a circle of radius *R*. Four variants of functions with different parameters *k*, taking values of 0.5, 0.75, 1, 1.5 were tested as a characteristic of background seismicity before the main shock:

$$r_k = \lg\left[\sum\nolimits_{t_0 - T \leq t \leq t_0} 10^{k(M(t)-M_m)}\right] - \lg S\,. \tag{2}$$

In this expression, *T* is the length of the time interval before the moment of the main shock t_0; $M(t)$ is the magnitude of the earthquake that occurred at time *t*; *S* is the area of a circle with a radius *R* defined by formula (1).

The expression in square brackets in formula (2) depending on the value of *k* has different meanings [*Kanamori, Anderson,* 1975]:

[1] *URL*: *http*://*seis-bykl.ru*/*modules*. *php*?*name*=*Data*&*da*=1 (accessed 25.12.2018).

k=0.5 is the ratio of the sums of the lengths of the earthquake faults before the main shock to the length of the main shock fault;

k=0.75 is the ratio of the total Benioff strain from earthquakes before the main shock to the strain resulting from the main shock;

k=1 is the ratio of the areas of the earthquake faults before the main shock to the area of the main shock fault;

k=1.5 is the ratio of the total scalar moment of earthquakes before the main shock to the scalar moment of the main shock.

The value r_k in expression (2) is the logarithm of the corresponding k characteristic, normalized to the area of a circle with radius R, from which background earthquakes are taken. The larger the r_k value, the higher is the preceding background seismic activity relative to the main shock.

We divide the set of r_k values into two noncrossing parts – $r_k<x$ and $r_k \geq x$. To detect the presence or absence of a background seismicity relation with the occurrence of a strong aftershock, we will consider the event $r_k>x$ as a precursor of a strong aftershock, and the $r_k \leq x$ event as an anti-precursor (precursor of the absence of a strong aftershock).

The traditional way of assessing the effectiveness of the precursor is the gain in probability [*Gusev*, 1974; *Aki*, 1993]:

$$PG(A, r_k>x) = P(A|r_k>x)/P(A), \tag{3}$$

where A denotes an event - the occurrence of a strong aftershock in the series; $P(A|r_k>x)$ is the probability of occurrence of a strong aftershock under the condition of $r_k>x$; $P(A)$ is the probability of occurrence of a strong aftershock. For an anti-precursor, the gain in probability is written by negating the conditions in brackets of expression (3): $PG(!A,r_k \leq x)=P(!A|r_k \leq x)/P(!A)$, where $!A$ is an event opposite to A (absence of strong aftershock in the series).

On the other hand, evaluation of the effectiveness of the considered precursor is the task of evaluating the effectiveness of a binary classification: there are two states - a strong aftershock is expected in the future series (first state) or not expected (second state). The standard method for evaluating the effectiveness of a binary classification is the *ROC*-diagram [*Rundle et al.*, 2011], which is a plot with horizontal axis representing the proportion of misclassifications when the precursor (FPR_1) and anti-precursor (FPR_2) conditions are fulfilled:

$$FPR_1=N(!A|\, r_k>x)/N(!A),\ FPR_2=N(A|r_k \leq x)/N(A); \tag{4}$$

on the vertical axis is the proportion of correct classifications when the conditions of the TPR_1 precursor and the TPR_2 anti-precursor are met:

$$TPR_1=N(A|\, r_k>x)/N(A),\ TPR_2 = N(!A|r_k \leq x)/N(!A). \tag{5}$$

In (4), (5) N (...) is the number of series satisfying the conditions indicated in brackets; $!A$ is an event opposite to A (absence of a strong aftershock). The optimal value of x can be determined by minimizing the loss function as $g= -1 - TPR + FPR$.

Both the gain in probability and the *ROC*-diagram consider “success” as the occurrence of a strong aftershock when the condition (precursor) is fulfilled, and “failure” or “false alarm” as the absence of a strong aftershock when the condition is met. In other words, “success” does not take into account the case when the condition is satisfied and there is no strong aftershock, and “failure”, in its turn, does not take into account the case of non-fulfillment of the condition when a strong aftershock occurs. Thus, these criteria of evaluating the precursor's effectiveness take into account only part of “successes” and part of “failures”, and as a result it is necessary to write out two separate expressions for gain in probability and for the *ROC*-diagram.

To eliminate this disadvantage, we consider the ratio of the sum of the shares of all "successes" to the sum of the shares of all "failures":

$$SFR(x)=[P(r_k > x|A) + P(r_k \leq x|!A)]/[P(r_k \leq x|A) + P(r_k > x|!A)]. \quad (6)$$

In the numerator (the sum of the shares of all "successes"): $P(r_k>x|A)$ is the probability that $r_k>x$, in the presence of a strong aftershock (A); $P(r_k \leq x|!A)$ is the probability that $r_k \leq x$ in the absence of a strong aftershock *(! A)*. The meaning of the terms in the denominator (the sum of the shares of all "failures") is obtained in a similar way. Note that

$$TPR_1+FPR_2=TPR_2+FPR_1=1.$$

If there is no connection between the precursor $r_k>x$ and the objective event A (the occurrence of a strong aftershock), then SFR=1; if this connection exists and it is positive, then SFR>1, and if it is negative, then SFR<1. The stronger the SFR deviates from 1, the more effective is the classification.

Calculation procedure

The left end of the time window T shifted to the past up to 5 years in increments of six months. At each iteration, according to formula (2), the values of r_k were calculated for k=0.5, 0.75, 1, 1.5 in a circle of radius R determined by formula (1); then, the SFR criterion value was calculated in accordance with expression (6); the values of the parameters cf, T and x, at which the maximum SFR was reached, were considered optimal. The results of the calculations are given in Table. 1, where x^* is the value when the maximum of the SFR criterion is reached; cf is the coefficient determining the radius of the circle R from which background earthquakes that occurred T years before the main shock are taken; $SFR^*= SFR(x^*)$ is the maximum value of the SFR criterion, determined by (6); TPR_1 (TPR_2) is share of true classifications: a strong aftershock is expected, and it occurs (a strong aftershock is not expected, and it does not occur), see formula (5); FPR_1 (FPR_2) is share of false classifications: a strong aftershock is expected, but it does not occur (a strong aftershock is not expected, but it occurs), see formula (4).

From the given in Table. 1 it follows that both for the Earth as a whole and for the considered seismically dangerous regions of Russia SFR>1.4 for any value of k.

Thus, it can be argued that there is a significant positive relationship between seismicity before the main shock and a strong aftershock ($M \geq \mathrm{E}[M_1]$) - the occurrence of strong aftershocks is more likely in places with high preceding seismicity.

For all regions, the most informative characteristic of preceding seismic activity is the ratio of the total scalar moment of earthquakes before the main shock to the scalar moment of the main shock – k=1.5 in formula (6). For the whole Earth and the regions under consideration, the maximum is $SFR \geq 1.4$. Note that for Baikal and Transbaikalia, as well as for the North Caucasus, the maximum SFR is reached both at k=1.5 and k=1. Nevertheless, based on the results obtained from global data, we will take k=1.5. for the most informative option for all regions.

For these regions of Russia, the studied relationship between the preceding seismicity and the occurrence of strong aftershocks is more pronounced than for the Earth as a whole. We attribute this to the existence of different seismic settings on Earth, determined by the features of seismogenesis, while in a particular region the seismic setting is usually of the same type.

Table 1. The maximum values of the *SFR* criterion calculated by the formula (6) for different values of *k*

k	SFR^*	x^*	cf	T (year)	TPR_1	FPR_1	TPR_2	FPR_2
Entire Earth, E[M_1]= –1.1								
0.5	1.47	–4.685	0.06	1.5	0.7	0.51	0.49	0.3
0.75	1.47	–4.653	0.04	3.5	0.66	0.47	0.53	0.34
1	1.49	–5.270	0.06	4	0.72	0.52	0.48	0.28
1.5	1.51	–6.598	0.03	1.5	0.72	0.52	0.48	0.28
Kamchatka and the Kuril Islands, E[M_1]= –0.9								
0.5	1.93	–4.224	0.09	0.5	0.8	0.48	0.52	0.2
0.75	2.01	–3.772	0.08	3.5	0.76	0.43	0.24	0.24
1	2.04	–5.157	0.08	0.500	0.82	0.48	0.52	0.17
1.5	2.29	–6.036	0.07	0.5	0.88	0.48	0.52	0.12
Baikal and Transbaikalia, E[M_1]= –1.5								
0.5	2.17	–3.620	0.04	1	0.37	0	1	0.63
0.75	2.17	–3.916	0.03	1.5	0.37	0	1	0.63
1	2.42	–4.502	0.03	1.5	0.47	0.06	0.94	0.53
1.5	2.42	–5.382	0.03	1	0.47	0.06	0.94	0.53
the North Caucasus, E[M_1]= –0.9								
0.5	2.13	–3.748	0.03	1	0.78	0.42	0.58	0.22
0.75	2.51	–4.245	0.03	1	0.72	0.29	0.71	0.28
1	2.60	–4.689	0.04	0.5	0.61	0.17	0.83	0.39
1.5	2.60	–5.657	0.04	0.5	0.61	0.17	0.83	0.39

The dependence of *SFR* on *x* (point of partitioning of the set of $r_{1.5}$ values) for the entire Earth is shown on the left in Fig. 1.) The share of true and false classifications can be estimated using the *ROC*-diagrams (Fig. 1, *on the right*). When k=1.5 and x>–6.598, the share of true classifications (a strong aftershock is expected and it occurs) is TPR_1=0.72, the share of false (a strong aftershock is expected and it does not occur) is FPR_1=0.52. When x≤–6.507, the share of true classifications (a strong aftershock is not expected, and it does not occur) is TPR_2=0.48, the share of false (a strong aftershock is not expected, and it occurs) is FPR_2=0.28. The areas under the *ROC*-diagrams coincide and are equal to 0.61.

The question arises, how to make a practical use of the discovered connection? As an answer to it, the next section discusses the calculation of the probability of occurrence of aftershocks with a magnitude not lower than the specified one.

Calculation of the probability of occurrence of aftershocks

To calculate the probability of an aftershock with a magnitude not lower than the specified one at different time intervals, we use the approach of P. Reasenberg and L. Jones [*Reasenber, Jones,* 1989], according to which we represent the aftershock process in the form of a superposition of Gutenberg – Richter [*Gutenberg, Richter,* 1954] and Omori – Utsu laws [*Utsu,* 1961]. Such a representation is correct if times and magnitudes of the aftershocks are independent (the Reasenberg – Jones hypothesis). The independence of times and magnitudes of aftershocks was proved at the global level for aftershock series of tectonic earthquakes with magnitude ≥6.5 [*Baranov, Shebalin,* 2019].

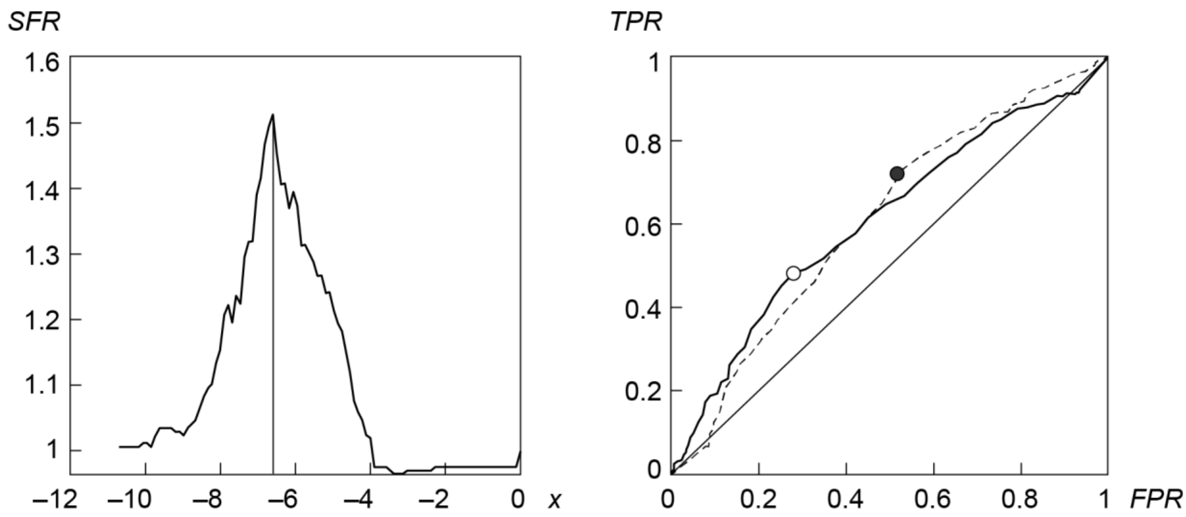

Fig. 1. *On the left:* dependence of the values of the *SFR* criterion determined by the formula (6) on x calculated for $r_{1.5}$ with the optimal values of the parameters cf=0.06, T=1.5 years. The vertical line corresponds to x*=argmax{*SFR*}. *On the right*: *ROC*-diagrams for $r_{1.5}>x$ (solid line, the loss function $g_1= -1-TPR+FPR$) and $r_{1.5}\leq x$ (dashed line, $g_2=-1-TPR+FPR$). White circle is the minimum of loss function g_1, black circle – of g_2. The diagonal (0, 0) (1, 1) corresponds to a random classification

If the number of representative aftershocks at the interval (t_{start}, T) is known, then according to the Omori – Utsu law [*Utsu*, 1961], the expected number of representative aftershocks at a certain time interval (t_0, t_1) has the form [*Baranov, Shebalin,* 2016; *Shebalin, Baranov,* 2017]:

$$\Lambda(t_0,t_1) = I(t_0,t_1)I(t_{start},T)/N(t_{start},T)\,. \tag{7}$$

Here t_{start} is the delay after the moment of the main shock that is necessary to eliminate the distortion of estimates due to the small number of aftershocks at the beginning of the series [*Smirnov*, *Ponomarev*, 2004; *Smirnov et al.*, 2010; *Narteau et al.*, 2009; *Holschneider et al.*, 2012]; $N(t_{start},T)$ is the number of aftershocks observed at the time interval (t_{start},T); I is the integral of the Omori – Utsu law with parameters c, p:

$$\Lambda(t_0,t_1) = I(t_0,t_1)I(t_{start},T)/N(t_{start},T),$$

$$I(\tau_0,\tau_1) = \int_{\tau_0}^{\tau_1} \frac{d\tau}{(\tau+c)^p} = \begin{cases} \ln\tau_1 - \ln\tau_0, & p=1 \\ \dfrac{(\tau_1+c)^{1-p} - (\tau_0+c)^{1-p}}{1-p}, & p\neq 1 \end{cases}$$

According to the Gutenberg – Richter law, the expected number of aftershocks with a magnitude not lower than M_0 is given by the expression

$$\Lambda(t_0,t_1,M_0) = \Lambda(t_0,t_1)10^{b(M_c-M_0)}, \tag{8}$$

where M_c is the representative magnitude; b is the recurrence plot slope.

Passing from the expected number of aftershocks to probability using the Poisson approximation, we obtain

$$P(t_0,t_1,M_0) = 1-\exp\{-\Lambda(t_0,t_1,M_0)\}\,. \tag{9}$$

Expressions (7) - (9) represent an averaged model of the probability of occurrence of aftershocks with a magnitude not lower than M_0 at the time interval (t_0, t_1). In [*Reasenberg*, *Jones*, 1989], the average values of the estimates for each considered series were used as parameters of the Gutenberg – Richter and Omori – Utsu laws. Such method of estimation is correct only in the case when the distribution of parameters is symmetric, which general case is not so. In addition, the parameters p and c of the Omori law are correlated with each other. Negative correlation of b and p values in interplate environments is observed from actual data [*Wang,* 1994; *Ogata, Guo,* 1997; *Gasperini, Lolli,* 2006]. The correlation of these parameters has physical reasons, as it is confirmed by laboratory experiments presented in [*Smirnov et al.*, 2019]. The authors of this work showed that, at failure along the formed fault, the relaxation parameter p increases with increasing axial stresses; the delay in the beginning of hyperbolic decay (parameter c in the Omori – Utsu law) decreases with increasing axial stresses and increases with increasing uniform compression pressure. In the same work, it was noted that the value of parameter b of the Gutenberg – Richter law inversely depends on the stress changes.

Parameter estimation

To estimate the averaged values of the parameters b, c, and p, we will use the approach implemented in [*Shebalin, Narteau*, 2017; *Baranov, Shebalin*, 2018b], and for estimating the parameter $N(t_{start}, T)$ - the law of aftershock repeatability [*Shebalin, Baranov, Dzeboev,* 2018]. In each region, we combine the considered series of aftershocks in a single set, replacing the magnitude of aftershocks M with the difference $M–M_m$ (relative magnitude) for each series and ordering the events according to increasing time relative to the main shock. Further, by the magnitude of the aftershock M we mean the difference $M=M–M_m$. We will evaluate the parameters by the set of series on the time interval (t_{start}, T). Using a set of series to determine the parameters allows to obtain estimates even for a small number of aftershock series, when simple averaging is meaningless.

The advantage of estimating the parameter b by a set of series is that the recurrence plot slope for all series is estimated in the area of high magnitudes. This allows to minimize the impact of the possible effect of a plot break due to accelerated plastic deformations in the earthquake source [*Vorobieva, Shebalin, Narteau,* 2016; *Shebalin, Baranov,* 2017].

For the whole Earth and the considered regions, we took t_{start}=0.05 days, T=365 days. According to the law of repeatability of aftershocks [*Shebalin, Baranov, Dzeboev,* 2018], the distribution of their number in the set of series is exponential:

$$P(\Lambda) = 1 – \exp(–\Lambda/\Lambda_0),$$

where $P(\Lambda)$ is the probability that the number of aftershocks is less than Λ; Λ_0 is the parameter whose estimate is the average number of aftershocks in a series.

Since the exponential distribution is asymmetric, than to estimate the expected number of aftershocks in the series, $N(t_{start},T)$ in (7), the median is preferable to the average, since the average in this case will give overestimated estimates. For the Earth as a whole and for the considered regions, excluding Baikal and Transbaikalia, $N(t_{start}, T)$ was calculated from the aftershocks that occurred during the time (t_{start}, T) with a relative magnitude not lower than –2, which is not lower than the representativeness of the corresponding catalogs. For Baikal and Transbaikalia, $N(t_{start}, T)$ was calculated from aftershocks with a relative magnitude not lower than –2.5 –2.5 (E[M_1]= –1.4), which is higher than the representativeness of the catalog.

Parameter b was estimated using the method presented in [*Bender*, 1983]. The range of relative magnitudes from –2 to –0.5 was used for the Earth as a whole, Kamchatka, and the Kuril Islands; for Baikal and Transbaikalia - from –2.5 to –0.5; for the North Caucasus - from –2 to –0.2. The left boundaries of the magnitude intervals do not exceed the values of the representative magnitudes of the used catalogs. Censorship on the right is introduced to prevent a possible bending of the repeatability plot due to the effect of finite volumes [*Romanowicz,* 1992].

An example of estimating the parameter b for the entire Earth without taking into account preceding activity is presented in Fig. 2; b values and 95-percent confidence intervals for the considered regions are given in Table. 2, where $N=N(t_{start}, T)$ is the median number of aftershocks in the series with a relative magnitude –2 or higher during the time . (t_{start}=0.05, T=365) days after the main shock in formula (7); E[M_1] is the average difference between the magnitudes of the strongest aftershocks and the main shock.

The parameters c and p of the Omori – Utsu law were estimated by the Bayes method [*Holschneider et al.,* 2012] according to the program of S.V. Baranov and P.N. Shebalin [2018c] with a uniform a priori distribution of c in the interval (0, T/2] and p in the interval [0.5, 1.5]. An example of estimating these parameters for the Earth as a whole without taking into account preceding activity is shown in Fig. 3; the values of c, p and 95-percent confidence intervals are given in Table. 2.

Similar estimation of the parameters was performed for the series with precursor ($r_{1.5}>x^*$) or anti-precursor ($r_{1.5}\leq x^*$) of a strong aftershock (see Table 1). The calculation results given in Table. 2 show that for the entire Earth and the three considered regions, the values of the parameters c and p of the Omori – Utsu law depend little on the presence or absence of a precursor of a strong aftershock, while the values of parameter b of the Gutenberg – Richter law and the median number of aftershocks in the series depend on the precursor.

According to the dynamic law of Bath [*Baranov, Shebalin,* 2018b], such differences in the parameters should lead to differences in the average values of the difference in magnitudes of the strongest aftershock and the main shock, which is confirmed by the values of E[M_1] given in the last column of Table. 2. This fact serves as an independent argument in favor of the relationship of seismic activity before the main shock with the occurrence of a strong aftershock in the future series.

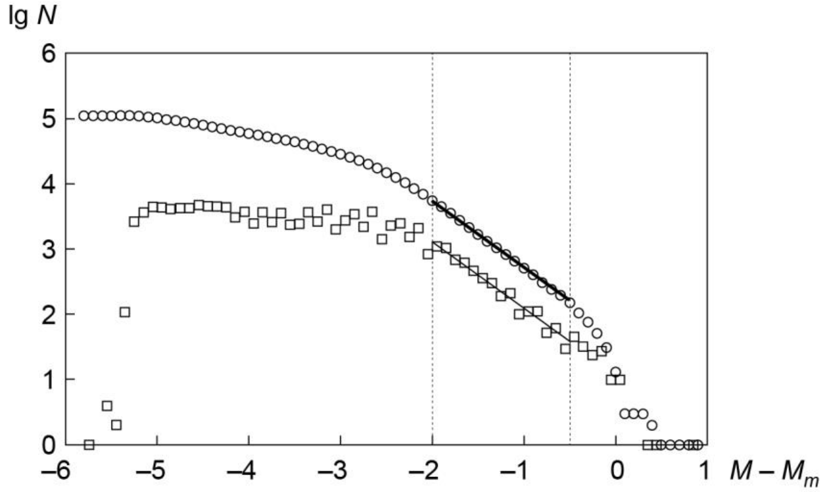

Fig. 2. Estimation of the b-value of the Gutenberg–Richter law for the Earth as a whole for a set of 786 aftershock series. Cumulative (bold line) and differential (thin line) magnitude-frequency curves for the relative magnitudes $M–M_m$. Circles denote the cumulative values, squares – differential ones. The dotted vertical lines – left (M_c= –2) and right (M_c= –0.5) mark the boundaries of the interval for estimating the b-value (the estimated b=1.02)

Table 2. Estimates of the model parameters (9) for all aftershock series and series with the presence of a precursor (anti-precursor) of a strong aftershock calculated for the entire Earth and seismically dangerous regions of Russia (Kamchatka and the Kuril Islands, Baikal and Transbaikalia, North Caucasus)

	N	b	c	p	E[M_1]
Entire Earth					
All series (718)	5	1.02 (0.98, 1.05)	0.12 (0.10, 0.15)	1.07 (1.06, 1.09)	–1.1
Precursor (445)	6	0.97 (0.94, 1.01)	0.14 (0.11, 0.18)	1.07 (1.05, 1.10)	–1
Anti-precursor (273)	4	1.14 (1.07, 1.21)	0.08 (0.04, 0.13)	1.07 (1.04, 1.11)	–1.3
Kamchatka and the Kuril Islands					
All series (69)	9	0.98 (0.84, 1.12)	0.15 (0.10, 0.21)	1.00 (0.96, 1.04)	–0.9
Precursor (49)	9	0.81 (0.74, 0.87)	0.16 (0.11, 0.22)	1.05 (1.01,1.07)	–0.7
Anti-precursor (20)	8.5	1.36 (1.19,1.55)	0.11 (0.02, 0.24)	1.16(1.07, 1.23)	–1.2
Baikal and Transbaikalia					
All series (36)	4.5	0.83 (0.74, 0.94)	0.02 (0, 0.06)	0.87 (0.81, 0.91)	–1.4
Precursor (10)	9	0.77 (0.56, 1.01)	0.02 (0, 0.08)	0.89 (0.82, 0.94)	–1.1
Anti-precursor (26)	3	0.90 (0.72, 1.10)	0.00 (0, 0.20)	0.82 (0.74, 0.93)	–1.6
the North Caucasus					
All series (42)	4	0.90 (0.83, 0.96)	0.13 (0.02, 0.32)	1.01 (0.92, 1.1)	–0.9
Precursor (15)	7	0.72 (0.62, 0.83)	0.08 (0.0, 0.26)	0.97 (0.86, 1.08)	–0.6
Anti-precursor (27)	2	0.92 (0.83, 1.01)	0.22 (0.02, 0.65)	1.06 (0.93, 1.21)	–1.1

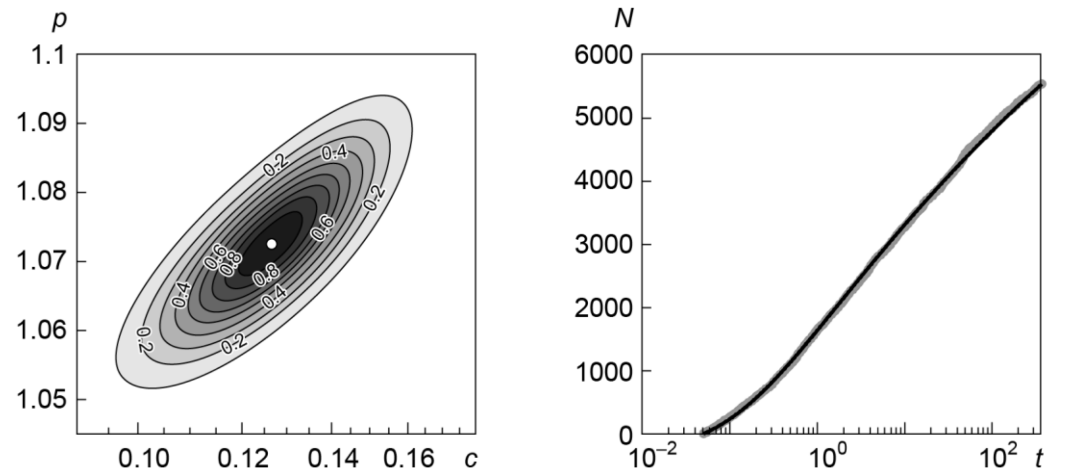

Fig. 3. Estimation of *c*- and *p*-parameters of Omori–Utsu law for the Earth as a whole for a set of 718 aftershock series. *On the left*: A posteriori distribution of Bayesian estimates of the *c* and *p*. The contours with markers show the quantile level lines, the white circle indicates the position of the maximum likelihood. *On the right*: distribution of the aftershock times. A black line indicates the empirical distribution for the set of 786 series, a thick gray line indicates the distribution according to the Omori–Utsu law with estimated parameters c=0.12 days, p=1.06

Results of probability calculation

To compare models (9) with and without precursor, the authors calculated probabilistic gains (3) from theoretical and observed values of probabilities. In this case, we used the parameters of model (9) for the considered regions given in Table. 2, and x* values from Table. 1. The obtained minimum values of probabilistic gains at the intervals (2^j, 365), where j=-5, -4, ...,7 days after the main shock are presented in Table. 3.

Table 3. The minimum values of probabilistic gains (3), calculated according to the model (9), taking into account the precursor of a strong aftershock and without it

Region	$PG(A, r_{1.5}>x^*)$		$PG(!A, r_{1.5}\le x^*)$	
	Theor.	Observed	Theor.	Observed
Entire Earth	1.17	1.17	1.03	1.02
Kamchatka and the Kuril Islands	1.23	1.22	1.09	1.17
Baikal and Transbaikalia	1.72	1.50	1.04	1.06
the North Caucasus	2.00	1.60	1.03	1.05

According to the performed calculations, the model with precursor is preferable to the model without it: the value of the gain in probability of occurrence of a strong aftershock $PG(A, r_{1.5}>x^*)$ for the whole Earth is not lower than 1.17, for the considered regions of Russia not lower than 1.23; the value of the gain in probability of the absence of a strong aftershock $PG(!A, r_{1.5}\le x^*)$ for the whole Earth and the considered regions of Russia is not lower than 1.03.

To verify compliance of the averaged model (9) with the real data, observed and theoretical probability were compared, calculated at a time interval (2^j, 365) days, where j=-5, -4, …, 7 after the main shock for aftershocks with magnitude E[M_1] or higher (Fig. 4, 5).

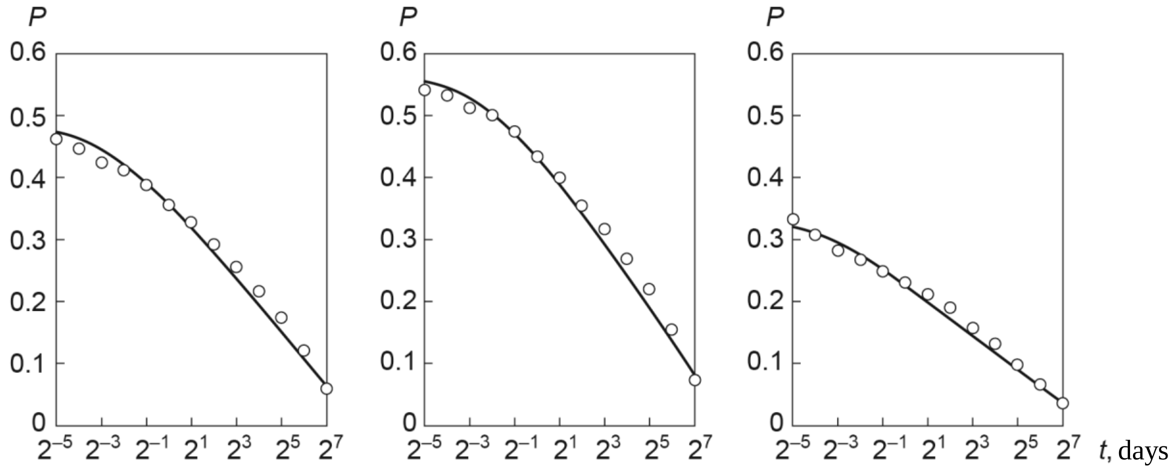

Fig. 4. Theoretical (solid line) and observed (circles) values of probabilities for the occurrence of at least one aftershock with a magnitude not less than E[M_1]≥ –1.1 to the time interval (2^j, 365), j=–5,–4, ..., 7 days after the mainshock. The calculations are presented (*left*) without taking into account the precursor; (*center*) in the presence of the precursor; (*right*) in the absence of the precursor for the entire Earth. The theoretical values are obtained using model (9) with the parameters from Table 2

The calculations showed that for all regions theoretical and observed probability values are well consistent: for the Earth as a whole (see Fig. 4), the absolute value of the average difference between the observed and model values is E≤0.006, the standard deviation is σ≤0.017; for the Kamchatka region and the Kuril Islands (see Fig. 5, I) – E≤0.013, σ≤0.027; for Baikal and Transbaikalia (see Fig. 5, II) – E≤0.013, σ≤0.045; for the North Caucasus (see Fig. 5, III) – E≤0.04, σ≤0.042. Thus, model (9) with the parameters given in Table. 1, 2, can be used in practice for estimating the probability of occurrence of repeated shocks with a magnitude not lower than the given one depending on time.

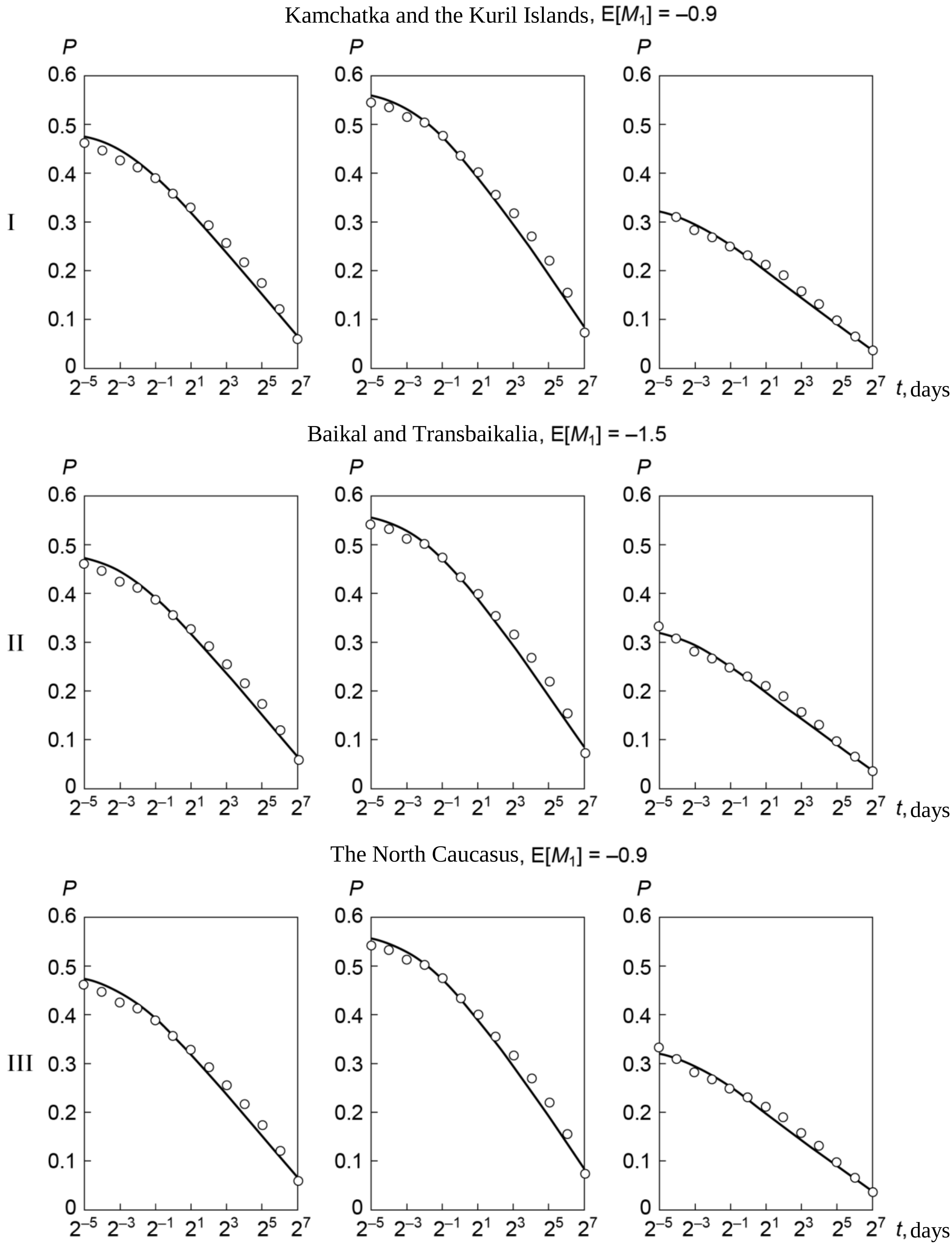

Fig. 5. The same as in Fig. 4 for three considered seismically dangerous regions of Russia: I – Kamchatka and the Kuril Islands, II – Baikal and Transbaikalia, III – the North Caucasus

Note that the authors considered all aftershock series with the number of aftershocks of at least 1 and the depth of the main shock up to 200 km. If additional constraints are imposed on the selection of series, for example, at least 5 aftershocks within 12 hours after the main shock, the values of the *SFR* criterion (6) for the whole Earth will be not lower than 1.8, and for the regions will be even greater. The values of the gains in probability of a model with a precursor relative to a model without a precursor will also increase.

At the same time, we believe that the task of assessing the danger of repeated shocks based on the information about aftershocks that have already occurred should be solved in a different way (see, for example, [*Shcherbakov, Zhuang, Ogata,* 2017; *Baranov, Pavlenko, Shebalin,* 2019]).

Conclusions

In this paper was tested the hypothesis that strong aftershocks are more likely to occur after main shocks confined to places with high background seismicity. Strong aftershock in the sense of Bath's law means that its magnitude not lower than the average difference of magnitudes of the strongest aftershocks and their mainshocks.

The study showed that the hypothesis is true for the entire Earth and for the three considered regions of Russia, regardless of the method of measuring preceding seismic activity. The values of probabilities of a strong aftershock occurrence for series with high preceding activity are higher than for all series, and the values of probabilities for all series are in turn higher than for series with low preceding activity. The ratio of the accumulated seismic moment of the preceding background seismicity to the seismic moment of the main shock characterizes the preceding seismicity in this task most informatively.

We obtained estimates of the parameters of the Reasenberg – Jones model for calculating the probability of an aftershock with a magnitude not lower than the threshold at the given time interval depending on the values of the background seismic activity preceding the main shock. The assessment was carried out using sets of aftershock series for the whole Earth and the three seismically dangerous regions of Russia (Kamchatka and the Kuril Islands, Baikal and Transbaikalia, the North Caucasus).

According to data for the entire Earth and seismically dangerous regions of Russia, we showed the correspondence of theoretical and observed values of the probabilities of the occurrence of at least one strong aftershock at different time intervals after the main shock. In this case, it is preferable to use the model taking into account the precursor than without it.

As a result of the research on global and regional data, it was established that seismicity preceding the main shock is one of the factors determining the occurrence of a strong aftershock.

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