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ON THE NONSTATIONARY PROCESSES IN GEOPHYSICAL MEDIA

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Abstract. In this methodological article we consider the effects that arise from the nonstationarity of the geospheres, namely, the magnetosphere, the atmosphere, and the lithosphere. The intensification of convection in the magnetosphere during the geomagnetic storm leads to the self-excitation of the electromagnetic ULF oscillations of the increasing frequency in the dusk sector. The paper gives a vivid example of observing oscillations of the increasing frequency at the mid-latitude Mondy station. It illustrates the exceptional complexity of nonstationary processes actually occurring in the magnetosphere. In the upper atmosphere (in the ionosphere), after the sunset, the source of ionization is turned off and the electron concentration begins to decrease. The nonstationarity of the medium consists in a monotonous decrease in the temperature of the atmosphere after the sunset, and it is manifested in the fact that the evolution of the electron density deviates noticeably from the prediction on the basis of a simple theory of recombination. This example is interesting in that it gives the key to understanding the known deviation of the aftershocks stream in the lithosphere from the simple Omori hyperbolic law. The marked analogy gives us an idea, firstly, to present the law in the form of a differential equation for the evolution of aftershocks and, secondly, gives us a non-trivial generalization of the Omori law, which takes into account the nonstationarity of the earthquake focus “cooling off” after the main shock. Methodologically, the above examples are supplemented by the hose MHD instability in the expanding solar corona, and the Jeans gravitational instability in the expanding Universe. The general conclusion is that it is necessary to carefully analyze the possible manifestations of the nonstationarity of the environment, even if the nonstationarity is smooth and, at first glance, is irrelevant.

Keywords: magnetosphere, ionosphere, lithosphere, ULF oscillations, earthquakes, recombination, deactivation, Omori law, hose instability, Jeans instability.

Introduction

Nonstationarity is observed everywhere, whether it be the expansion of the Universe, the evolution of stars, global warming on the Earth and so on. And it always arouses the research interest and stimulates the search for its common causes and specific mechanisms. In this methodological article, however, a different aspect of the problem will be discussed. We will leave aside the question of the origin of nonstationarity of the environment and focus on the fact how nonstationarity manifests itself, i.e. how it affects physical processes in one or another natural environment.

At first sight, it may seem that when analyzing sufficiently fast processes one can neglect the smooth changes in the parameters of the medium over time, i.e. the medium can be considered as quasi-stationary that is not always the case. In this regard, recall a classic example of the Jeans gravitational instability that leads to the formation of galaxies [Jeans, 1902]. In the unperturbed state the Jeans Universe is homogeneous and static, and small density perturbations in it grow exponentially when a certain criterion is met. The theory is not quite satisfactory, since it contains the so-called gravitational paradox (see, for example, [Kipper, 1962]). The paradox is removed when considering the expansion of the Universe according to the Hubble’s law, but density perturbations increase in a power-law manner over time, and not exponentially. It is seen that the nonstationarity of the Universe significantly affects the star formation rate.

Another example of this kind is related to a hose MHD instability in the solar corona that constantly expands into the interplanetary space in the form of the solar wind [Parker, 1965]. The corona expansion is neglected when analyzing a hose instability (see, for example, [Guglielmi, Pokhotelov, 1996]). Meanwhile, the mentioned example of Jeans instability suggests that the corona expansion, perhaps, should be taken into account when analyzing the hose instability. Since this issue is directly related to geophysics, we will return to it below in the “Discussion” section. But before that, we consider the characteristic effects arising due to nonstationarity of three geospheres – the magnetosphere, the atmosphere and the lithosphere (see sections with the corresponding names). The magnetosphere is highly nonstationary during the geomagnetic storm when the so-called ring current in the radiation belt is amplified. Quasiperiodic modulation of the atmosphere temperature occurs due to daily rotation of the Earth. It is clear from general considerations that the lithosphere is also nonstationary, but in this case it is more difficult to distinguish a certain physical parameter for simple analysis. We will focus on aftershocks which evolution, apparently, is one of the clearest manifestations of the lithosphere nonstationarity in the earthquake focus.

The magnetosphere

During the geomagnetic storm, when the magnetosphere nonstationarity appears most clearly, the ULF electromagnetic oscillations are usually observed in the dusk sector which frequency monotonically increases from about hertz to several hertz for about half an hour [Guglielmi, Troitskaya, 1973]. These oscillations were earlier discovered by V.A. Troitskaya [Troitskaya, 1961], and called *Intervals of Pulsations of Diminishing Period (IPDP)*. It was later found [Zolotukhina, 1979, 1981; Kangas, Guglielmi, Pokhotelov, 1998] the existence of two species of *IPDP* – injection and convective (Fig. 1).

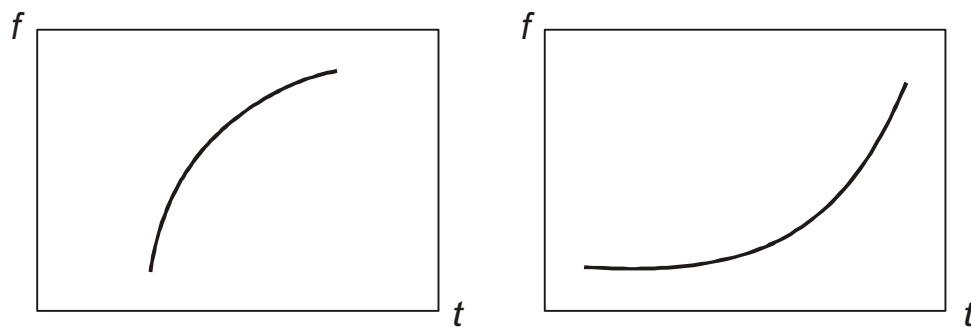


Fig. 1. Schematic representation of the dynamic spectra of *IPDP* type of the ULF oscillations: (*left*) injection and (*right*) convective species

It is obvious that nonstationarity of oscillation spectrum reflects nonstationarity of the magnetosphere during the storm. Moreover, we will now see, that the basic properties of *IPDP*, namely, self-excitation in the 1Hz range, localization mainly in the dusk sector, monotonous frequency increase and existence of two oscillation types are a direct manifestation of distribution nonstationarity of energetic protons in the outer radiation belt in the main phase of the geomagnetic storm.

Recall that the main phase of the storm is characterized by a significant decrease in the geomagnetic field, that is associated with the amplification of so-called ring current formed by the radiation belt particles drifting in azimuth around the Earth [Nishida, 1980]. Amplification of the ring current can occur, firstly, as a result of pulsed injection of «fresh» particles into the magnetosphere and, secondly, due to the amplification of the magnetosphere convec-

tion and the associated movement of the ring current along the radius (closer to the Earth). Apparently, observation of injection and convective *IPDP* species reflects the existence of two named nonstationarity mechanisms of the ring current.

Injection of new particles into the ring current as well as movement of the ring current closer to the Earth due to magnetospheric convection amplification leads to formation of non-equilibrium velocity distribution of energetic protons— non-monotonic and generally speaking anisotropic [Guglielmi, Pokhotelov, 1996]. Non-equilibrium distribution can be unstable, besides oscillation frequency ω is close to the so-called resonance frequency $\omega_R - \omega \approx \omega_R$. If $c_A \ll v_p$, then resonance frequency is approximately equal to

$$\omega_R \approx (c_A/v_p)\Omega_p. \quad (1)$$

Here $\Omega_p = eB/m_p c$ is a proton gyrofrequency; $c_A = B/\sqrt{4\pi\rho}$ is Alfvén velocity; v_p is a characteristic proton velocity of the ring current; e is an elementary electric charge; m_p is a proton mass; c is a velocity of light; ρ is a plasma density; B is a magnitude of the geomagnetic field at the equator of magnetic shell with parameter L , and $B \propto L^{-3}$. According to current estimates, ω_R corresponds to *IPDP* frequency range.

Imagine that at the moment t_0 in the vicinity of midnight meridian there was a pulsed injection of energetic protons into the magnetosphere from the geomagnetic tail. The longitude φ will be counted from midnight to the west. For clarity, let the compact proton cloud has at longitude $\varphi=0$ a stable velocity distribution of particles (Maxwell equilibrium distribution). After injection ($t > t_0$) the protons drift westwards; however, drift velocity of each particle is proportional to its energy. As a result, the cloud spreads along the longitude and function of particle distribution becomes non-equilibrium (non-monotonic). The dusk sector ($\varphi \sim \pi/2$) initially receives more energetic particles and, accordingly, oscillation increment has a peak at lower frequencies, and then less energetic particles arrive. As a result, oscillation frequency monotonously increases. According to the formula (1) we obtain the estimate

$$\omega \propto \sqrt{t - t_0}. \quad (2)$$

The situation with convective *IPDP* is somewhat more complicated. We will briefly discuss the general idea of interpretation. First, assume that the magnetosphere is stationary. The quiet solar wind with frozen in it the interplanetary magnetic field excites a large-scale stationary electric field E in the magnetosphere, that for simplicity we assume homogeneous in plane of geomagnetic equator. Field E is directed from the morning side to the evening. The so-called electric field of corotation E_{\oplus} is imposed on it, i.e. the field of unipolar induction associated with the daily rotation of the Earth. In the equatorial plane the field is directed towards the Earth's center and decreases with distance from it: $E_{\oplus} \propto L^{-2}$. Gradient drift in heterogeneous geomagnetic field is imposed on electric drift of charged particles in total field of convection and corotation. Velocity of gradient drift, unlike electric drift, depends on particle energy. This circumstance makes it difficult to analyze the drift trajectories (see, for example, [Nishida, 1980; Guglielmi, Pokhotelov, 1996]). If we ignore the details that are certainly important for understanding how non-equilibrium (non-monotonic) energy distribution of protons is formed in the dusk sector as a result of electric and gradient drift, then provided information is enough to understand in general terms the mechanism of frequency increase of the *IPDP* convective oscillations.

Indeed, on the evening meridian $\varphi = \pi/2$ in equatorial plane there is an exceptional point L_* , where the corotation field E_{\oplus} and the convection field E are equal in magnitude and

opposite in direction. The mentioned above non-monotonic energy distribution of protons that leads to the self-excitation of oscillation is formed right in the vicinity of L_* . It is easy to verify that $L_* \propto E^{-1/2}$. As the dynamic pressure of the solar wind increases, the field E increases with time, the point L_* moves closer to the Earth and oscillation frequency increases:

$$\omega \propto [E(t)]^{3/2}. \quad (3)$$

Here we used the formula (1), condition of first adiabatic invariant conservation ($v_p \propto L^{-3/2}$) and so-called gyrofrequency model of radial distribution of plasma density ($\rho \propto L^{-3}$).

In conclusion of this section we give a striking example of *IPDP* observation at Mondy mid-latitude station (Fig. 2).

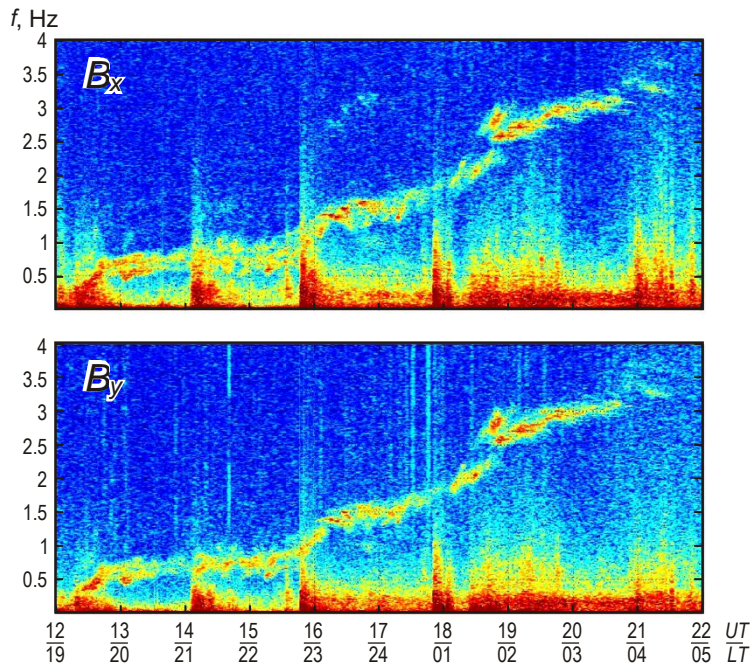


Fig. 2. Oscillations of increasing frequency according to observations of 17.03.2013 at Mondy station (51.6 N, 100.9 E, $L=2.2$) [Potapov *et al.*, 2016]

The given example is interesting due to the fact that it illustrates the exceptional complexity of processes actually occurring in the magnetosphere. Apparently, we observe the manifestation of increasing convection, against which occasionally occur strong impulse injections of hot protons. Detailed analysis of this event is presented in [Potapov *et al.*, 2016].

The atmosphere

In this section, we will briefly discuss the nonstationarity of the upper ionized layers of the atmosphere – the ionosphere. It is about the cooling-down of the atmosphere after the sunset. We confine ourselves only by giving one good example illustrating the need to take into account the nonstationarity of the ionosphere while analyzing processes occurring in it. This will help us to understand the logic of reasoning in the next section when studying far more complex processes in the lithosphere that lead to earthquakes.

After the sunset the ionization source turns off and concentration of electrons in the upper ionized layers of the atmosphere begins to decrease due to the recombination of electrons and positively charged ions. As an elementary example, let us consider a radiative recombination proceeding according to the scheme



where O_2 is an oxygen molecule; O_2^+ is an oxygen ion; e^- is an electron; $\hbar\omega$ is photon (see for example [Ginzburg, 1960]). A pair of charges disappears as a result of recombination, and a neutral molecule and a photon appear. Let n_+ (n_-) be a density of positive (negative) charges, and $n = (n_+ + n_-)/2$. Then the recombination equation has a form

$$dn/dt = -\alpha n_+ n_-, \quad (5)$$

where α is a recombination coefficient. Due to plasma quasi-neutrality $n_+ = n_-$, so the general solution of the equation (5) has the form

$$n(t) = n_0 (1 + \alpha n_0 t)^{-1}. \quad (6)$$

In deriving formula (6) we assumed that $\alpha = \text{const}$, but is it really true? It turns out that our assumption of the constancy of the recombination coefficient is incorrect. The point is in the following. The recombination coefficient α depends on the temperature T . After the sunset, the ionosphere begins to cool off and temperature T decreases. In other words, $\alpha(t) = \alpha[T(t)]$ nonstationarity occurs. Therefore, the law of evolution of the density of charged particles due to radiative recombination significantly changes:

$$n(t) = n_0 \left[1 + n_0 \int_0^t \alpha(t') dt' \right]^{-1}. \quad (7)$$

Let us pay attention to an interesting analogy: modification of the evolution law of the ionosphere due to temperature decrease after the sunset resembles remotely modification of Jeans gravitational nonstationarity due to the Universe expansion after the Big Bang. Nonstationarity of the medium significantly affects the decrease in the density of charged particles in the first case and the growth of gas density perturbations in the second.

The lithosphere

Apart from the details, the upper atmosphere and the magnetosphere are characterized by lability, fast reaction to variations in solar activity and, accordingly, rapid variability of parameters. In contrast to it, the non-stationary processes in the massive lithosphere in general proceed rather slowly. A noticeable acceleration of the processes occurs in the earthquake focus just before the main shock and immediately after it, that can be seen from characteristic manifestations in the form of foreshocks and aftershocks. But even in these cases, nonstationarity of parameters of hard shell is mainly latent, unlike the nonstationarity of gas and magnetic shells of the planet. It is especially difficult to detect and control the nonstationarity of parameters of geological environment before the main shock, although foreshocks and other manifestations of nonstationarity indirectly indicate the impending catastrophe (see, for example, the review [Guglielmi, 2015] and references cited in it).

Under these circumstances we will focus on aftershocks – shocks that occur in the epicentral zone after the main shock – for which the empirical Omori law was reliably established [Omori, 1894]. In the context of this article, it is interesting that formally the Omori law completely coincides with the law (6) of radiative recombination in plasma. This gives us the idea, firstly, to introduce the law in the form of differential equation of evolution of aftershocks

$$dn/dt + \sigma n^2 = 0 \quad (8)$$

and, secondly, it gives us a non-trivial generalization of the Omori law, that takes into account the nonstationarity of the earthquake focus, “cooling off” after the main shock:

$$n(t) = n_0 \left[1 + n_0 \int_0^t \sigma(t') dt' \right]^{-1}. \quad (9)$$

Here $n(t)$ is a frequency of aftershocks; n_0 is an initial condition. Phenomenological parameter $\sigma(t)$, included in the evolution equation (8) and describing the current state of rocks is naturally to call deactivation coefficient of the focus [Guglielmi, 2016; Guglielmi, 2017].

The further development of the theory is suggested by the remote analogy between state variations of rocks in the focus and climate variations of the Earth. Suppose, there is a quasi-equilibrium state $\bar{\sigma}(\varphi)$, where φ is yet an unknown set of internal parameters of the focus, generally speaking, dependent on time. Then as the basis of relaxation theory of focus deactivation can be assumed the equation

$$\frac{d\sigma}{dt} = \frac{\bar{\sigma}(\varphi) - \sigma}{\tau} + \xi(t), \quad (10)$$

similar to the one used for describing the average temperature of the earth's surface [Byalko, 2012]. Here τ is a characteristic time of σ approaching to the equilibrium state $\bar{\sigma}$. Function $\xi(t)$ simulates the external influences on the focus. Integrating (10), we obtain

$$\sigma(t) = \left\{ \sigma_0 + \int_0^t [\xi(t') + \tau^{-1} \bar{\sigma}(\varphi(t'))] \exp(t' / \tau) dt' \right\} \exp(-t / \tau). \quad (11)$$

The choice of function $\xi(t)$ requires a special consideration. External disturbances are, for example, round-the-world seismic echo of the main shock [Zotov *et al.*, 2018], spheroidal oscillations of the Earth [Guglielmi, Zotov, Zavyalov, 2014] and, possibly, electromagnetic fields of natural or artificial origin [Buchachenko, 2014]. In each particular case, impulse, periodic or stochastic function $\xi(t)$ can be chosen.

Discussion

If the observer moves in a stationary, but inhomogeneous medium, then an illusion of nonstationarity may occur. Movement of the observer in non-uniform stationary light flux can serve as an example. This is a too trivial example, but let us recall that such an optical illusion earlier generated geocentric system of the world. (However, according to the sociological survey of VCIOM (RPORC), many still believe that the Sun revolves around the Earth.) A more sophisticated example is a uniform straight motion of an observer in the wave field of whistling atmospheric. Analysis leads to the paradoxical conclusion that, contrary to common sense, the group velocity of a whistling atmospheric does not depend on rate of motion of observer [Guglielmi, 1963].

Let us, however, proceed to the discussion of really nonstationarity media and start with an analysis of the hose instability in the expanding solar corona. The hose instability leads to variations in the interplanetary magnetic field (IMF), and this, in its turn, significantly affects the dynamics of the Earth's magnetosphere. In particular, IMF controls the excitation mode of the ULF oscillations of the geomagnetic field (see, for example, [Guglielmi, Potapov, Dovbnya, 2015]).

In stationary plasma, the dispersion equation for long-wave MHD perturbations has the form

$$\omega^2 = c_A^2 k_{\parallel}^2 \left[1 + \frac{8\pi N}{B^2} (T_{\perp} + T_{\parallel}) \right], \quad (12)$$

where ω is a frequency; k_{\parallel} is a longitudinal component of wave vector; N is an electron concentration; B is a homogeneous external magnetic field; T_{\parallel} (T_{\perp}) is a longitudinal (transverse) temperature. Plasma is considered to be collisionless, isothermal and anisotropic ($T_{\parallel} \neq T_{\perp}$). It is seen that when

$$T_{\parallel} > T_{\perp} + \frac{B^2}{8\pi N} \quad (13)$$

an aperiodic instability develops in plasma (a more detailed analysis is given in monograph [Guglielmi, Pokhotelov, 1996]). However, generally speaking, criterion (13) cannot be used to assess the stability of the solar corona, since it does not take into account nonstationarity of medium parameters. It is quite clear, that nonstationarity should also be taken into account when analyzing not only a hose, but also other types of instability of the expanding corona.

Let us explain how nonstationarity arises. First, consider the expansion of the corona in the system of fixed stars. The magnitude of IMF decreases with increasing distance from the Sun according to the law that follows from the equation $\text{div} \mathbf{B} = 0$ and freezing-in condition $\text{rot} \mathbf{U} \times \mathbf{B} = 0$ under simplifying assumption, that $U = \text{const}$, –

$$B(R) \propto R^{-2} [1 + (\Omega_{\odot} R/U)^2]^{1/2}, \quad (14)$$

where R is a heliocentric distance; Ω_{\odot} is an angular rotational velocity of the Sun; U is a radial expansion velocity of the corona.

Now we need to go in the accompanying reference frame moving along with the selected plasma volume. For this purpose we make a substitution $R = Ut$ and get $B(t) \propto t^{-2}$ at $R \ll U/\Omega_{\odot}$, i.e. at relatively short distances from the Sun. Quite similarly we get $N \propto t^{-2}$, $c_A \propto t^{-1}$ and $T_{\parallel}/T_{\perp} \propto t^2$. We see that all the parameters included in formulas (12) and (13) are time-dependent. Thus, the standard theory of the hose instability is inapplicable to the solar corona.

Manifestation of nonstationarity in the magnetosphere was examined in the form of frequency modulation of the ULF sporadic oscillations of *IPDP*. Modulation in the form of relatively smooth increase in the oscillation frequency displays transient processes in the radiation belt during the main phase of the magnetic storm. In this regard it is worth noting that sharp upward jumps in the carrier frequencies of the permanent ULF oscillations are observed during sudden commencement of the magnetic storm. Modulation of this type reflects a sharp magnetosphere compression by interplanetary shock wave advancing the flare stream of the solar wind [Kangas, Guglielmi, Pokhotelov, 1998].

Manifestations of ionosphere nonstationarity are extremely diverse. Above was given a typical example that gave us an idea of the role of nonstationarity of geological environment in the formation of averaged evolution of aftershock flow; the idea is presented in the form of the equation (8) and the formula (9).

In conclusion of this section, we will briefly discuss question of how this result can be used to state the inverse problem of physics of the earthquake focus cooling off after the main shock. Let us consider the Volterra integral equation of the first kind

$$\int_0^t K(t, t') \sigma(t') dt' = g(t), \quad (15)$$

where $\sigma(t)$ is a required function, and $g(t) = [n_0 n(t)]^{-1} [n_0 - n(t)]$ is a known function. The inverse problem consists in determining the deactivation coefficient $\sigma(t)$ according to the observation of aftershock frequency $n(t)$. Shape of kernel $K(t, t')$ should be the subject of further study. But even if the trivial kernel $K=1$ is used, then even in this simplest case the inverse

problem has a substantial sense. Indeed, if $K=1$, then the equation (15) directly follows from the aftershock equation (8)¹.

Conclusion

General conclusion that can be drawn from the above is that it is useful and instructive to analyze together examples of similar phenomena borrowed from various fields of natural science. Sometimes already simple associations suggest the formulation of new questions, as it follows from our comparison of the expanding Universe and the expanding solar corona. The question of apparent nonstationarity mentioned by us is interesting from the point of view of an observer, moving in non-uniform stationary medium, in particular, in non-uniform stationary wave field. Physically meaningful and promising is the problem of relationship between the magnetosphere nonstationarity and spectrum nonstationarity of the ULF oscillations.

However, in our opinion, the most significant result was obtained on the basis of formal analogy between recombination of charges of opposite sign in the ionosphere and deactivation of faults in the lithosphere. On this path, we managed to outline contours of relaxation theory of aftershocks and formulate the inverse problem of physics of the earthquake focus. Like any new knowledge, our result still has the character of version and hypothesis and only time will show how productive is the proposed version in the general context of the earthquake physics.

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¹ For preliminary results of search for $\sigma(t)$ in this formulation, see [Guglielmi et al., 2017].

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